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Mathematica® as a Tool for Studying Mathematics in Distance Learning Environment

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Abstract

Distance education in Malaysia was first pioneered by Universiti Sains Malaysia (USM) in the 1970s and has since undergone a fundamental transformation from primarily print-based to multi-media and e-learning. The rapid spread of information and communication technology (ICT) has changed the landscape of the learning environment that includes student-centered learning, a new emphasis on experimentation, exploration and computation and the use of the internet. The driving force behind these changes has been the evolving technology. This article reports the qualitative studies based on the authors' experience, observations and personal interactions with distance learners in an attempt to introduce the Computer Algebra System (CAS) in the teaching and learning of Mathematics. The principal aim is to explore student perceptions towards using *Mathematica* CAS as the computational tool to learn mathematics at a distance. The ultimate aim will be to integrate *Mathematica* into the course content as a support to enhance the adult student learning experience. This ongoing initiative has its successes and failures that depend on many factors including student readiness, accessibility of the programme and designing the *Mathematica*-based learning materials as discussed in the article.

Keywords : Distance Learning, learning environment, Mathematics, *Mathematica*

Abstrak

Pendidikan jarak jauh di Malaysia terlebih dahulu dipelopori oleh Universiti Sains Malaysia (USM) pada tahun 1970-an dan sejak itu telah mengalami satu transformasi asas terutamanya daripada pembelajaran berasaskan cetakan kepada pembelajaran berasaskan multimedia dan e-pembelajaran. Penyebaran maklumat dan teknologi komunikasi (ICT) yang pantas telah mengubah landskap persekitaran pembelajaran termasuklah pembelajaran yang bertumpukan pelajar, penekanan baru kepada eksperimentasi, eksplorasi serta pengkomputeran dan penggunaan internet. Daya penggerak di sebalik semua perubahan ini berpunca daripada teknologi yang sentiasa berkembang. Artikel ini melaporkan kajian kualitatif berdasarkan pengalaman, pemerhatian dan interaksi penulis secara peribadi dengan pelajar jarak jauh dalam usaha untuk memperkenalkan *Computer Algebra System (CAS)* dalam pengajaran dan pembelajaran Matematik. Tujuan utama adalah untuk meneroka persepsi pelajar ke arah menggunakan *Mathematica CAS* sebagai alat pengkomputeran bagi mempelajari matematik secara jarak jauh. Matlamat terakhir pastinya untuk menyepadukan *Mathematica* ke dalam isi kandungan kursus sebagai satu sokongan untuk mempertingkatkan pengalaman belajar dalam kalangan pelajar dewasa. Inisiatif yang masih berterusan ini ada kelebihan dan kekurangannya, bergantung kepada banyak faktor termasuk kesediaan pelajar, ketercapaian program dan mereka bentuk bahan pembelajaran berasaskan *Mathematica* seperti yang dibincangkan dalam artikel ini.

Kata kunci: Pembelajaran Jarak Jauh, persekitaran pembelajaran, Matematik, *Mathematica*

Introduction

The study of algorithms useful for computer algebra systems is known as computer algebra**.** This software program popularly known as computer algebra system (CAS) has the core functionality of manipulating mathematical expressions in symbolic form. The history of Computer Algebra Systems can be traced back to the early 1970s and evolved out of research into artificial intelligence. Amongst the first popular CAS were Reduce, Derive and Macsyma whilst the current market leaders are *Mathematica*, MATLAB and Maple.

The 21st century students are well exposed to computers and such system would provide a revolutionary way of presenting scientific knowledge and principles to the students. Furthermore, with the availability of affordable and powerful computers, the use of CAS in many disciplines and Mathematics education is becoming increasingly important and widespread. The benefits of using CAS in teaching and learning Mathematics have been reported in many conferences and literatures (Artigue, 2002; Pierce and Stacey, 2004; Lavicza, 2006). With CAS, the teaching of Mathematics becomes more interesting and students are more motivated to learn Mathematics (Pierce, 2005). Many studies have looked at the effects of using CAS on students' mathematical skills and their understanding of concepts. One example is a case study on the comparison between the achievements of group taught using CAS and group with traditional teaching (Meel, 1998). From various studies reported, it provides strong evidence that proper use of CAS enables students to develop deeper conceptual understanding. Undoubtedly, the knowledge gained in the use of CAS can be an asset for students' learning and future postgraduate studies.

In this work, we decided to use *Mathematica* in our endeavor to introduce CAS to distance learners who are taking Mathematics courses at the School of Distance Education (SDE), Universiti Sains Malaysia (USM). We would like to explore the full potential of the software program and to study the response of the adult students towards its usefulness in their distance learning environment. Some of the important issues need to be addressed are, how to create curricula and dynamic teaching and learning environments into which *Mathematica* is integrated as well as how to design innovative pedagogical methodologies, techniques, and materials based on symbolic computation for distance learning environment.

A report by Wellins (1991) in the first editorial journal of *Mathematica* in Education and Research stated that there were four components of *Mathematica*'s potential for changing education:

1. Active involvement of students in learning.
2. Experimentation as a means of understanding mathematical concepts.
3. Visualisation of mathematical processes.
4. Access of students to real-world problems.

In the early years of its introduction, integrating CAS in Mathematics education has not been without controversy due to perceptions of automation and anxiety over the diminishing of basics mathematical skills amongst students. Although the use of *Mathematica* CAS has now become quite common in Science and Mathematics in the traditional face to face learning, their use in the distance learning environment is still narrow and relatively sparse. One particular example of highly successful use was reported in the NetMath distance-learning programme offered at the University of Illinois. The NetMath model for distance learning is a blend of the interactive nature of the *Mathematica* software and the internet (Uhl, Davis and Porta, 2011).

In the School of Mathematical Sciences, USM, there are courses on MATLAB and *Mathematica* being offered to the full time campus students. However, the School of Distance Education through its Mathematics programme has yet to offer any computing courses to their students. Currently, USM has the campus-wide site license for the latest version of *Mathematica 8.0* and running on standard Windows OS. The program may be installed in any machine connected to the USMnet and also includes limited home use for faculty members. *Mathematica* training workshops and seminars were conducted for the on campus staff and students. For the benefit of new users, Wolfram has also made available in a web-based introductory tutorial for *Mathematica* to facilitate student learning in Science, Technology, Engineering, and Mathematics with this cutting-edge computational software engine.

Motivations and Initiatives to Introduce *Mathematica* at the SDE

The Mathematics distance education programme offered by the SDE makes use of primarily print-based self-instructional modules and text books, E-learning portal, live video conferencing over broadband network connections for wired Regional Centers, three weeks face-to-face Intensive Course on the main campus, CD-based video lectures and e-mail/telephone. The Mathematics courses are traditional in nature with pen-paper approaches of memorizing rules and techniques to solve typical undergraduate Mathematics problems.

With CAS already widely available and becoming ubiquitous, distance learners should no longer be constrained by pen-paper approach in learning of Mathematics. They should also be equipped with the necessary computer skills and expose to the state of art of computation tool to thrive the challenging life in a lifelong learning culture.

Though there are other CAS available like Maple and MATLAB in the market, *Mathematica* is chosen simply because of its functionality and language structure. Its programming language is very structured and highly adaptive to a variety of applications, both technical and theoretical. On a closer look, the basic structure and syntax of *Mathematica* reflects the structure and syntax of Mathematics. We believe that learning of *Mathematica* and learning to apply and program it would provide a valuable exercise in thinking mathematically. These are some of the reasons that motivate us to introduce the programme to distance learners.

Basically there are three level of uses of *Mathematica* as an assistance to mathematical learning:

1. Presentation tool.
2. Tool to develop more conceptual understanding.
3. Technical tool performing technical task.

Initially we started using *Mathematica* as a presentation tool that was used merely for demonstrating mathematical ideas and generating graphs used in video conferencing sessions and face-to-face tutorials during Intensive Course. As an aid to visualisation, it is a great tool to convey information. Another stronger feature of *Mathematica* is its ability to manipulate the graphics along with calculations and text, all in one environment.

In a later development, in order to facilitate the use of *Mathematica* as a learning tool, some materials on the topics of differential equations were prepared in *Mathematica* notebook format and posted to a personal author's website and the school e-learning portal (Moodle). The students were advised to download the free version of *Mathematica* Reader to enable them to read the document**.**

The third level is the black box level and it offers little pedagogical value for distance learners. However, for students who decided to do final year Mathematics project, they were encouraged to use *Mathematica.* They were trained on a personal basis and had to take work leave to attend on campus workshop.

In an attempt to broaden students experience with *Mathematica*, a hands-on workshop was organized for interested students during the Intensive Course period. The basic of *Mathematica* commands were introduced during the short training and pre-prepared *Mathematica* notebooks were given to enable students to familiarise with the commands and system. Initially, they just learnt how to execute the computations and interpret results generated by *Mathematica*. Explorations were sometimes applied to introduce topics as well as consolidate concepts. With guided examples, students began to explore concepts and patterns of Mathematics as well as to use it as a tool in solving simple differential equations they met in the printed modules.

With this approach, it can be argued that as *Mathematica* does more lower-level symbolic operations, higher-level thinking and deeper understanding of the output generated by *Mathematica* are expected from the students. We began to realize the big challenge ahead in choosing the appropriate mathematical problems and pedagogical strategies carefully in the light of using *Mathematica* for real learning. Moving beyond the routine mathematical manipulations, or translating conventional mathematical words and symbols into *Mathematica* commands, to explaining working, justifying reasoning or interpreting results, requires students to verbalize their mathematical thinking thus making the approach more student-centered learning. For these to happen, the distant students should be familiar to the technology and software before they use them as tools for learning.

Creating Engaging Content with *Mathematica*

Technology-enhanced learning which was formerly known as Computer Assisted Learning (CAL) and later e-learning is still a hot topic in education. In the early days of e-learning, instructors simply upload their teaching material online. The content which lacks interactivity will limit understanding of the material. In the distance learning environment, interaction is one of the important instructional element and is the key to success. There are three types of interactions as identified by Moore and Kearsley (1996) namely student-instructor, student-student and student-content. Part of the learning process includes how students interact with the content presented for e-learning. There is strong evidence to suggest that highly interactive material may contribute to more learning.

Interactivity needs familiarity in the applications software, but most course managers are non-programmer. Moreover, authoring interactive material in HTML5, Flash or JAVA is hard and time consuming. Recently Wolfram Research launched a Computable Document Format (CDF) which is designed to facilitate course authors and publishers to create and incorporate knowledge applications into documents.

CDFs can be authored using *Mathematica* 8 which is distributed for free using the Wolfram CDF Player. The CDF enables course authors to embed interactive charts, diagrams and graphics into their e-Modules and can provide a unique capability for interactive learning. It will turn “lifeless learning materials” into ones with highly interactive and illustrate concepts. As shown in Figure 1, taken from the Pearson's Calculus CDF-enhanced e-book, it illustrates interactively that continuous function does not necessarily imply differentiable function.

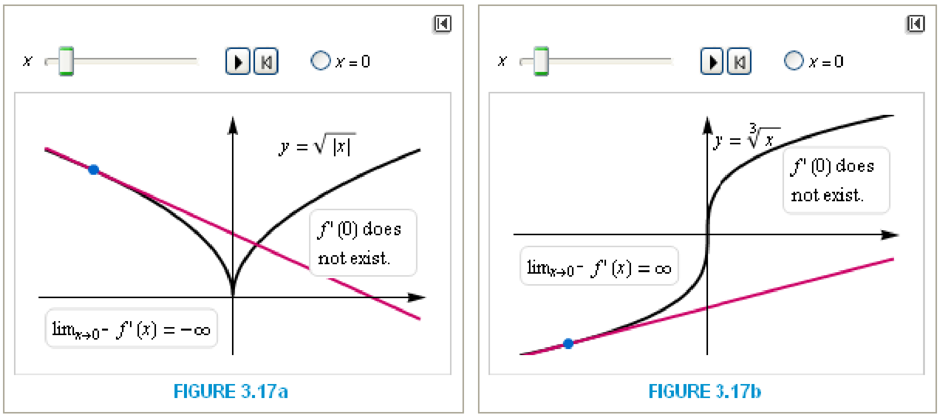

Figure 1 Illustrating continuous function does not imply differentiable function

The web*Mathematica* is one of the earlier technologies developed by Wolfram Research which augments *Mathematica* main capabilities and features over the web. Instructors need some knowledge of web*Mathematica* to create a dynamic interactive web-based learning. With web*Mathematica*, students will be able to make remote access to software resources for dynamic interactive mathematical computations and visualization for distance learning. Without web*Mathematica* technology, instructor has to be well versed with complex CGI or Java programming to make a webpage mathematically active (Figure 2).

We have developed web-based Mathematics instructions on the topics of Bezier and B-Spline Curves to enable students study Mathematics in an e-learning mode (Mohamad Faisal, Majid and Piah, 2004).

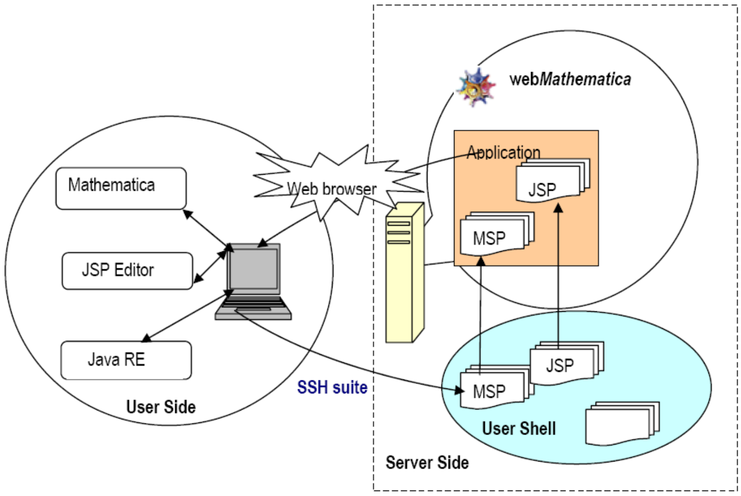

Figure 2 web*Mathematica* architechure

Another product developed by Wolfram Research based on *Mathematica* computational engine is *Wolfram|Alpha* (Figure 3). It is free, internet-based and brings a new type of knowledge representation to the web world. It computes answers to specific questions using its built-in knowledge base and algorithms. *Wolfram|Alpha* is suitable for inquiry-based learning but has not yet been exposed to the SDE students.

In addition to web deployment, other applications of Wolfram|Alpha for mobile learning offers exciting possibilities for distance learners. Mobile devices such as iPhone and iPad shown in Figure 4 can be used to support distance learning Mathematics.

Providing students with such new technologies could boost their engagement and participation in collaborative learning that would allow them to create work and share with other students and faculty members.

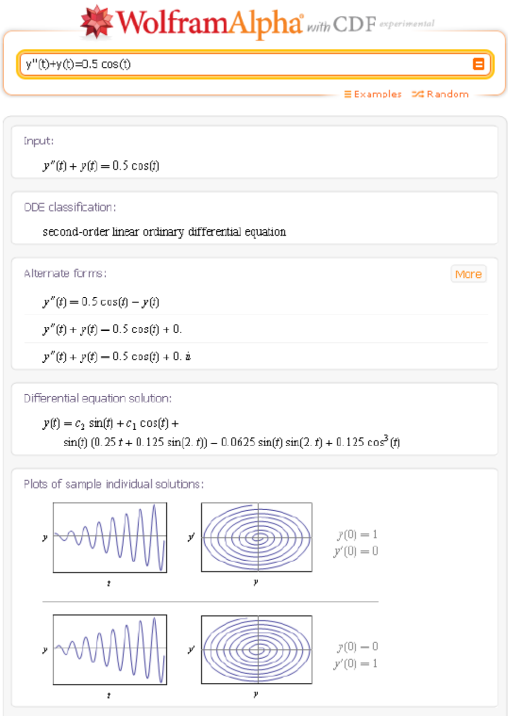

Figure 3 Resonance solution of $y''(t) + y(t) = 0.5\cos(t)$

Figure 4 iPhone and iPad applications in mobile learning with Wolfram|Alpha

Feedback and Response from the SDE Students

Introducing computer aided instructions with *Mathematica* program to adult distance learners has given rise to a special challenge. Based on workshop observations and student interviews, we have found that :

1. All students who attended the workshop had no basic knowledge about *Mathematica* or other CAS.
2. Some students experienced difficulties in basic computer usage and file management such as saving and locating files.
3. Students who enjoyed using *Mathematica* particularly liked symbolic computations like differentiation, integration and factorizing algebra and rechecked the answers with pen and paper work.
4. Students expressed moderate enthusiasm for the workshop and made enquiries on how to obtain the program.
5. Highly constrained syntax and unexpected outputs from *Mathematica* were quoted by students as to be the distractions in learning Mathematics. This is expected from the inexperienced and amateur users.
6. Students commented that, provided they knew what commands to use, using *Mathematica* would come in handy to check anwers and plot quick graphs.
7. The workshop was viewed by some students as useful for their Final Year project and wished they could practice with it at their respective Regional Centres.
8. While experimenting with *Mathematica* by manipulating mathematical expressions, the input entered sometimes produced unexpected results with error messages. The students were at a deadlock when determining the problems and were a bit frustrated.

In spite of some difficulties, general comments from students who attended the workshop during 2005-2007 Intensive Course, believed that *Mathematica* would be a useful tool in learning Mathematics. The response to one particular question i.e. whether all students who are taking JIM 213-Differential Equations and JIM 311-Vector Analysis courses should attend *Mathematica* workshop during the Intensive Course did gain mixed views from the students. The tight schedule during the Intensive Course had prevented some of them from attending the workshop. Others keep wondering about the usefulness of *Mathematica*, whether it might assist them at all in the Continuous Examination held during the Intensive Course. A clear pattern was evident - highly potential students might use *Mathematica* to augment their mathematical learning through investigations, pattern observations, work checkings, as a trouble-shooter of complicated problems and also use it in their Final Year project.

Anecdotal evidence clearly indicates that generally, adult distance learners at the SDE were reluctant, worried and felt intimidated when using computers in learning Mathematics. Being the digital immigrants, they need support, assurance and motivation when learning Mathematics with *Mathematica*. These observations are consistent with studies by Kirkwood (2003).

We realised that gradual and gentle introduction of *Mathematica* to adult distance learners is thus very important so as not to intimidate them with the strict *Mathematica* syntax and progamming.

Problems and Issues

The university has licensed *Mathematica* with premier service which provides additional benefits such as Home Use, Workbench, web*Mathematica* and many more. *Mathematica* is licensed in 2 ways: *Stand Alone Licenses* and *Network Concurrent Licensing*.

Currently, all Regional Centres throughout Malaysia that serve as the centre for distance learning do not provide computer lab facilities. Over a remote connection, this will of course requires more bandwidth. One solution in computing would be to use Virtual Network Computing (VNC) which is a graphical desktop sharing system that uses the Remote Frame Buffer (RFB) protocol to remotely control another computer. The network traffic for a VNC connection is smoother and much more responsive than for a remote X connection with *Mathematica*'s highly interactive notebook features. Ideally, the SDE should has arranged the *Mathematica* to be installed as a stand-alone version for students. Cost savings of the bulk purchases of licenses for Mathematics distance learners will be another important factor to be decided by the SDE.

Two other issues arised are: "Should *Mathematica* be taught as a separate subject as being practiced on campus mode?" and "Should *Mathematica* be incorporated into the teaching of an existing Calculus or Differential Equations courses?"

Conclusion: Directions for Future Investigations

This work constitutes a first document on what is currently being done with regard to the use of CAS-based technology in Mathematics distance learning at SDE. The use of *Mathematica* in distance learning environment is still relatively new but its extended use in future is imminent. Using *Mathematica* and the *Mathematica*-based interactive resources for on- and off-line learning effectively by distance learners demands careful planning and commitment.

The authors believe that Mathematics curriculum at SDE should take advantage of the availability of *Mathematica* on USM campus to redesign the teaching-learning model and moving towards an active student-centered learning process. On the institution side, besides focusing on making *Mathematica* accessible to distance learning students, well-trained lecturers and the sustainable of investments in the technology are equally important.

For future investigation, it will be interesting to know how the distance learners would be able to cope with *Mathematica*, and if indeed those using *Mathematica* in their learning would help them in learning and improving their grades.

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